S+NuOPT Technical Note

Applying Mixed Integer Programming (MIP) to Portfolio Optimization Problems Using S+NuOPT™

Many practical optimization problems encountered in finance, such as portfolio or trading optimization, require the formulation of integer constraints. Without the availability of mixed integer programming functionality such as provided by S+NuOPT, the only methods left are heuristic in nature, and the solutions they generate are very often sub-optimal.

What is MIP?

Mathematical programming problems in which some of the variables are restricted to integer values are called mixed-integer programming problems (MIP). Solving MIP problems is usually a computationally intensive task, in some cases beyond the abilities of even the most sophisticated commercial computers. MIP is useful for describing key characteristics of many real-world finance problems involving optimization problems with discrete constraints.

MILP/MIQP stands for Mixed Integer Linear Programming/Mixed Integer Quadratic Programming and refers to mathematical programming problems with a linear/quadratic objective function where some or all variables are restricted to be integers. Unlike standard linear/quadratic programming (LP/QP) where all variables are continuous, MILP/MIQP requires special solving algorithms such as branch-and-bound where a sequence of sub-problems is solved and optimality identified afterwards. In the following descriptions, MIP refers to both MILP and MIQP unless otherwise stated.

Why is MIP Important?

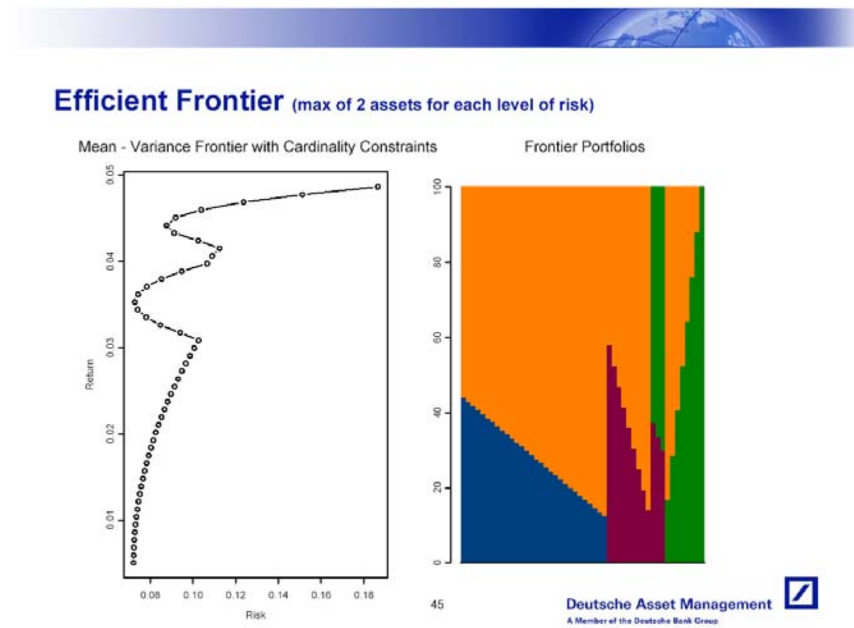
Many practical optimization problems encountered in finance, such as portfolio or trading optimization, require the formulation of integer constraints. Without the availability of MIP functionality, the only methods left are heuristic in nature. The disadvantages of solving optimization problems via heuristic methods include: 1) there is no commonly recognized implementation standard; 2) there is no defined theory for comparing different algorithms; 3) it is uncertain how close a solution is to the true optimal; and 4) heuristic procedures are expensive to implement in many cases. These issues are addressed in S+NuOPT's implementation of MIP where the optimality is solved for in one single step rather than being approximated via iterations.

Applications of MIP in Quantitative Finance

There is a variety of applications of MIP in the finance industry:

- Basket selection: given an initial portfolio, select basket of trades given that only a maximum number of trades are allowed (along with other turnover constraints).
- Cardinality Constraints (number of names constraints): given the asset universe, portfolio managers often need to limit the total number of holdings (both long and short) in their final portfolio.
- Buy In Threshold Constraints: some assets can only be purchased or sold at certain minimum levels, or overly-small holdings or trades may need to be excluded in an optimum portfolio.

- Round lots: restrictions defining the basic investment unit. For instance, investors are only allowed to make transactions in integer multiples of these round lots.



NuOPT Webcast presentation by Dr. Bernd Scherer,
Head of Investment Solutions Europe, Deutsche Asset Management, February 2003[1]

- Lower partial moments optimization: Instead of using variance as a risk measure, it is sometimes desirable to optimize on downside deviation. The ability to declare and use integer variables makes it easy to formulate the problem elegantly and therefore solve more efficiently.

Why use S+NuOPT?
S+NuOPT's MIP functionality has a tested track record and is being used by leading hedge funds, buy-side and sell-side managers to find optimal portfolio strategies in universes with thousands of assets and large number of constraints.

MIP is now recognized as the most natural way to solve optimization problems with integer constraints and S+NuOPT provides straightforward capabilities to apply these techniques.

S+NuOPT supports the following optimization methods: 1) simplex method for linear programming and mixed integer programming (MILP) models, for very large scale problems; and 2) active set method for convex quadratic programming models and mixed integer quadratic programming (MIQP) models.

[1] Archived and available for download under http://www.insightful.com/company/events.asp